

MTE-04

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

ELEMENTARY ALGEBRA

(Valid from 1st January, 2019 to 31st December, 2019)

It is compulsory to submit the assignment before filling in the exam form.



**School of Sciences
Indira Gandhi National Open University
Maidan Garhi
New Delhi-110068
(2019)**

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

- 1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:

NAME:

ADDRESS:

.....

.....

COURSE CODE:

COURSE TITLE:

ASSIGNMENT NO.:

STUDY CENTRE: **DATE:**

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

- 2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
- 3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
- 4) Your answers should be precise.
- 5) While solving problems, clearly indicate which part of which question is being solved.
- 6) **This assignment is valid only upto December, 2019.** If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.
- 7) It is compulsory to submit the assignment before filling in the exam form.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

ASSIGNMENT

(To be done after studying Blocks 1 and 2.)

Course Code: MTE-04
Assignment Code: MTE-04/TMA/2019
Maximum Marks: 100

- 1) Which of the following statements are true? Justify your answers. (This means that if you think a statement is false, give a short proof or an example that shows it is false. If it is true, give a short proof for saying so. For instance, to show that ‘{1, padma, blue} is a set’ is true, you need to say that this is true because it is a well-defined collection of 3 objects.)
- i) For any two sets A and B, $A \cap B^c = A \setminus B$.
- ii) The matrix $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ is singular.
- iii) The contrapositive of ‘ $\exists y \in \mathbb{Z}$ such that P(y) is true’ is ‘ $\exists x \in \mathbb{Z}$ such that P(x) is true’.
- iv) The system $2x - 3y = 1$ and $6y - 4x + 2 = 0$ has a unique solution.
- v) If $x, y \in \mathbb{C}$ such that $x^2 = y$ and $y^2 = x$, then $x = y = 1$.
- vi) $a \geq b \Leftrightarrow -a \leq -b$ is an absolute inequality.
- vii) If $A = \emptyset, B = \{1, 2\}, C = \{-1, -2\}$, then $A \times B \times C$ has 4 elements.
- viii) The argument of $1 + \sqrt{3}i$ is $\frac{\pi}{3}$.
- ix) A linear equation over \mathbb{R} can have at most one root in $\mathbb{C} \setminus \mathbb{R}$.
- x) $|x_1 - x_2| = |x_1| - |x_2| \forall x_1, x_2 \in \mathbb{R}$. (20)
- 2) a) Prove that $2^n > 1 + n\sqrt{2^{n-1}} \forall n > 2$, using the inequalities of Unit 6. (4)
- b) For $a_1, \dots, a_n \in \mathbb{R}, a_1 < a_2 < \dots < a_n$, show that
- $$\frac{n}{a_1 - a_0} + \frac{n-1}{a_2 - a_1} + \dots + \frac{1}{a_n - a_{n-1}} \geq \sum_{k=1}^n \frac{k^2}{a_k} \quad (8)$$
- c) Use Weirstrass’ inequalities to prove that
- $$\left(\sum_{i=1}^n \frac{1}{\sqrt{i}} \right) \leq \frac{1}{\sqrt{n!}} \prod_{i=2}^n (\sqrt{i-1}) + 2 \left(\sum_{i=2}^n \frac{1}{\sqrt{i}} \right) \quad (3)$$
- 3) a) If ω is a cube root of unity, show that $(1 - \omega + \omega^2)(1 + \omega - \omega^2) = 4$. (2)
- b) Use De Moivre’s theorem to show that $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$. (3)
- c) Describe the geometric, polar and exponential representations of $(-5i + 2)^{-1}$. (5)
- 4) a) Let $A = \{x \in \mathbb{Z} \mid x \text{ is a multiple of } 5\}$ and $B = \{x \in \mathbb{Z} \mid x \text{ is a divisor of } 20\}$. Represent A, B and $A^c \cap B$ by the listing method and in a Venn diagram. (5)

- b) Prove that $(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$ for any two sets A and B in a universal set U. (5)
5. a) Apply Cardano's method for finding the roots of $2x^3 + 3x^2 - 8x - 12 = 0$. (7)
- b) Find the polynomial equation over \mathbb{R} of lowest degree which is satisfied by $1 - i$ and $3 + 2i$. (3)
6. a) Give a direct proof, as well as a proof by contradiction, of the following statement: ' $A \cap B \subseteq A \cup B$ for any two sets A and B .' (4)
- b) Use the principle of induction to prove the statement given in Q.2(a). (4)
- c) Write the converse of the statement: 'If p and q are the only roots of a polynomial f over \mathbb{C} , then $\deg f = 2$.' (2)
7. a) Solve the linear system $x + 2y = 4, 3x - y = 2$ (I) by substitution. If $4x + y = c, ax + by = 3$ is a system having the same solution set as (I), find a, b, c. (5)
- b) If $\begin{vmatrix} 2 & 3 & 7 \\ 1 & x & -2 \\ 0 & 2 & x \end{vmatrix} = 5$, find x. (2)
- c) Apply Cramer's rule to solve the following system of equations:
$$\begin{aligned} 2x_1 + x_2 + x_3 &= 4 \\ x_1 - x_2 + 2x_3 &= 2 \\ 3x_1 - 2x_2 - x_3 &= 0 \end{aligned}$$
 (5)
- d) Consider the equation $E \equiv 5x - 2y = 3$. Write down equations E_1, E_2, E_3 , respectively so that
- E and E_1 are inconsistent;
 - E and E_2 have a unique solution;
 - E and E_3 have infinitely many solutions. (3)
8. 3 major projects P_1, P_2, P_3 are being funded by 3 voluntary agencies V_1, V_2, V_3 . V_1, V_2, V_3 are willing to pay Rs. 8,000/-, Rs. 4,000/- and Rs. 2, 000/-, respectively per person on the project P_1 ; Rs. 4,000/-, Rs. 3,000/- and Rs. 4, 000/- respectively per person on P_2 ; and Rs. 3,000/-, Rs. 5,000/-, Rs. 8,000/- respectively on the project P_3 . Further, the amount that V_1, V_2, V_3 have kept aside for paying people on these projects is Rs. 2,17,000/-, Rs. 1,42,000/- and Rs. 1,32,000/- respectively. How many people should each project employ so that the total money available is utilised? (10)